

Show ALL work

- 1) Find all solutions (real or complex) of $x^2 - 2x + 5 = 0$

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{4 - 20}}{2} \\&= \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = \frac{2(1 \pm 2i)}{2} = 1 \pm 2i\end{aligned}$$

- 2) Write $\frac{3-4i}{2+i}$ in the form $a + bi$

$$\begin{aligned}\frac{3-4i}{2+i} \cdot \frac{2-i}{2-i} &= \frac{6 - 3i - 8i + 4i^2}{4 - i^2} = \frac{6 - 11i - 4}{5} \\&= \frac{2 - 11i}{5} = \frac{2}{5} - \frac{11}{5}i\end{aligned}$$

- 3) Find all real solutions of the equation.

a) $x^4 - 8x^2 - 9 = 0$

$$u = x^2$$

$$u^2 - 8u - 9 = 0$$

$$(u-9)(u+1) = 0$$

$$u = 9 \text{ or } u = -1$$

$$x^2 = 9 \text{ or } x^2 = -1$$

$$\textcircled{x = \pm 3} \quad \times$$

b) $\sqrt{2x+3} = 2x+1$

Square

$$2x+3 = (2x+1)^2$$

$$2x+3 = 4x^2+4x+1$$

$$0 = 4x^2+2x-2$$

$$0 = 2(2x^2+x-1)$$

$$0 = 2(2x-1)(x+1)$$

$$x = \frac{1}{2} \text{ OR } x = -1$$

check

$$x = \frac{1}{2}$$

$$\sqrt{4} = 2$$

✓

$$x = -1$$

$$\sqrt{1} = -1$$

X

ANSWER

$$\boxed{x = \frac{1}{2}}$$

c) $|3x-2| = 4$

$$3x-2=4 \text{ OR } 3x-2=-4$$

$$3x=6$$

$$3x=-2$$

$$x=2 \text{ OR }$$

$$x = -\frac{2}{3}$$

4) Solve the inequality. Graph the solution set on the real number line.

a) $x^2 + 4x - 12 > 0$

$$(x+6)(x-2) > 0$$



$$x = -7 \quad - \cdot - = +$$

$$x = 0 \quad + \cdot - = -$$

$$x = 3 \quad + \cdot + = +$$

$$x < -6 \text{ OR } x > 2$$

$$(-\infty, -6) \cup (2, \infty)$$



b) $|2x-1| \leq 3$

$$-3 \leq 2x-1 \leq 3$$

$$-2 \leq 2x \leq 4$$

$$-1 \leq x \leq 2$$

$$[-1, 2]$$

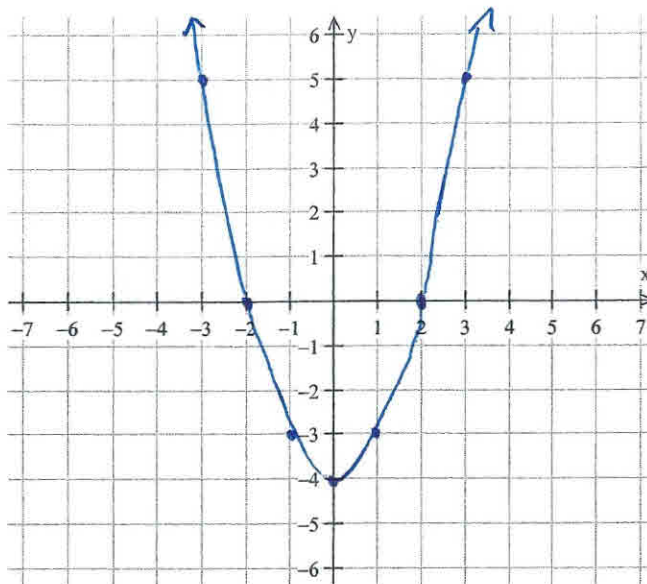


5) Find the intercepts and graph by plotting points $y = x^2 - 4$.

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

$$\text{not if } x=0 \\ y=-4$$

$$\text{if } y=0 \\ 0 = x^2 - 4 \\ 4 = x^2 \\ \pm 2 = x$$



6) Let $P_1 = (2, -3)$ and $P_2 = (4, 5)$

a) Find the distance between P_1 and P_2

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 2)^2 + (5 - (-3))^2} = \sqrt{2^2 + 8^2} = \sqrt{68}$$

b) Find the midpoint of the line segment containing P_1 and P_2 .

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{2 + 4}{2}, \frac{-3 + 5}{2} \right) = (3, 1)$$

c) What is the slope of the line containing P_1 and P_2 ?

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{4 - 2} = \frac{8}{2} = 4$$

d) What is the equation of the line through P_1 and P_2 ?

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= 4(x - 2) \\ y + 3 &= 4x - 8 \\ y &= 4x - 11 \end{aligned}$$

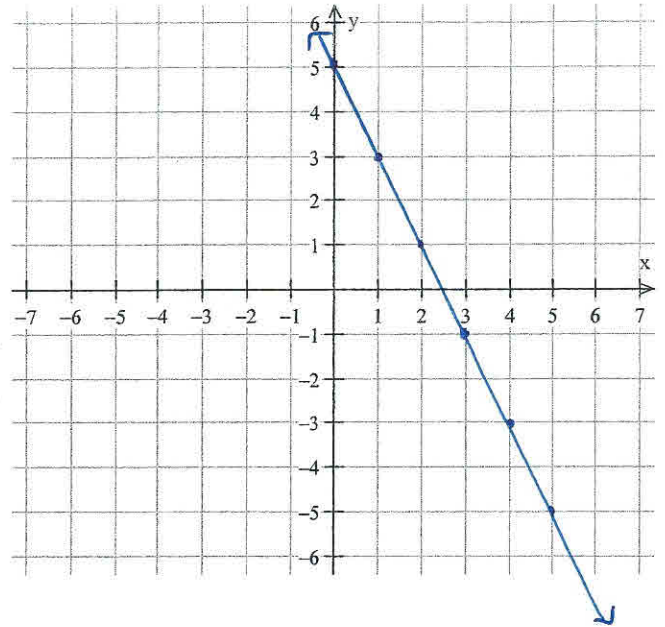
e) What is the equation of a line through P_1 with undefined slope?

$$\underline{x = 2}$$

7) What is the slope of any line perpendicular to $2x + y = 6$? $\frac{1}{2}$

$\downarrow$
 $y = 6 - 2x$
slope -2
need negative reciprocal

8) Graph the line containing the point $(1, 3)$ and having slope -2 .



9) What is the center and radius of the circle $(x - 2)^2 + (y + 1)^2 = 5$?

center $(2, -1)$
radius $\sqrt{5}$

10) Find the standard equation of the circle with general equation

$$x^2 + y^2 - 8x + 2y + 8 = 0$$

$$x^2 - 8x + y^2 + 2y = -8$$

$$x^2 - 8x + 16 + y^2 + 2y + 1 = -8 + 16 + 1$$

$$(x - 4)^2 + (y + 1)^2 = 9$$

$$\left(\frac{1}{2}b\right)^2$$